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# Wide-range tunability, thermal locking, and mode-crossing effects in Kerr optical frequency combs

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**Abstract.** We theoretically and experimentally investigate some effects related to the Kerr optical frequency comb generation, using a millimeter-size magnesium fluoride ultrahigh quality disk resonator. We show that the Kerr comb tunability can be extremely wide in the Turing pattern (or primary comb) regime, with an intermodal frequency that can be tuned from 4 to 229 multiple free spectral ranges (corresponding to a frequency spacing ranging from 24 GHz to 1.35 THz). We also discuss the role played by thermal locking while pumping the resonator, as well as the effect of modal crossing when broadband combs are generated. © 2014 Society of Photo-Optical Instrumentation Engineers (SPIE) [DOI: [10.1117/1.OE.53.12.122602](https://doi.org/10.1117/1.OE.53.12.122602)]

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## 1 Introduction

Monolithic ultrahigh quality ( $Q$ ) factor optical resonators with small mode volumes have been used for the study of various nonlinear optical phenomena in the continuous-wave (CW) low pump power regime. Kerr nonlinearity-based hyperparametric oscillations have been investigated along that line.<sup>1,2</sup> The interplay of Kerr nonlinearity and group-velocity dispersion in these solid-state resonators further results in the formation of interesting optical frequency combs through a cascaded four-wave mixing process. Since the demonstration of the first extended Kerr frequency comb generation in an on-chip silica microtoroid resonator,<sup>3</sup> an intensive amount of work has been carried out to further understand this phenomenon. This interest has been driven by the fact that compared with mode-locked laser-based frequency combs, Kerr combs feature a compact size and consume less power.

Up to now, Kerr frequency combs have been demonstrated in various materials and resonator geometries. They usually include fused silica microtoroids,<sup>3,4</sup> microrods,<sup>5-7</sup> silica microdisks,<sup>8</sup> on-chip microring structures using high-index glass,<sup>9</sup> silicon nitride,<sup>10-15</sup> aluminum nitride,<sup>16</sup> and disk resonators made of magnesium fluoride<sup>12,17-22</sup> and calcium fluoride crystals.<sup>20,23-26</sup> The integrated high- $Q$  microring structure enables full chip-scale integration with waveguides. On the other hand, crystalline disk resonators can feature ultrahigh  $Q$  factors ( $>10^9$ ). Thus, they help to reduce the required pump power for comb generation, and they have the potential to produce low-phase noise microwave signals. Theoretically, better understanding of Kerr frequency combs has also been emerging. A modal expansion approach was recently derived,<sup>27,28</sup> and more recently, a normalized Lugiato-Lefever formalism has been applied on simulating microresonator-based Kerr combs.<sup>29-31</sup> As a result,

the formation of Turing patterns, solitons, and their stability analysis have been studied.<sup>19,32-34</sup>

From the application point of view, the potential application of Kerr frequency combs has been demonstrated in many fields, such as multiwavelength high-speed coherent data transmission with advanced modulation formats,<sup>13</sup> ultra-short pulse generation,<sup>15,22</sup> low-phase noise microwave generation,<sup>8,20,23</sup> and so on. Considering the stable and spectrally pure microwave signal, many advanced techniques have been recently applied for stabilizing combs, such as self-injection locking,<sup>7</sup> interleaved electro-optical comb stabilization,<sup>5,35</sup> Pound-Drever-Hall locking (PDH),<sup>20,23</sup> and atomic Rb transitions referencing.<sup>20</sup> However, many studies have shown that the formulation of full Kerr frequency combs may involve many stages and could feature multiple radio frequency (RF) beatnotes and relatively high phase noise.<sup>8,12,14</sup> Recently, studies propose that the initial stage of Kerr combs, the primary combs, or Turing patterns, is strongly phase-locked and could be ideal for a secondary frequency reference,<sup>21</sup> similar to hyperparametric oscillations.<sup>2,36</sup>

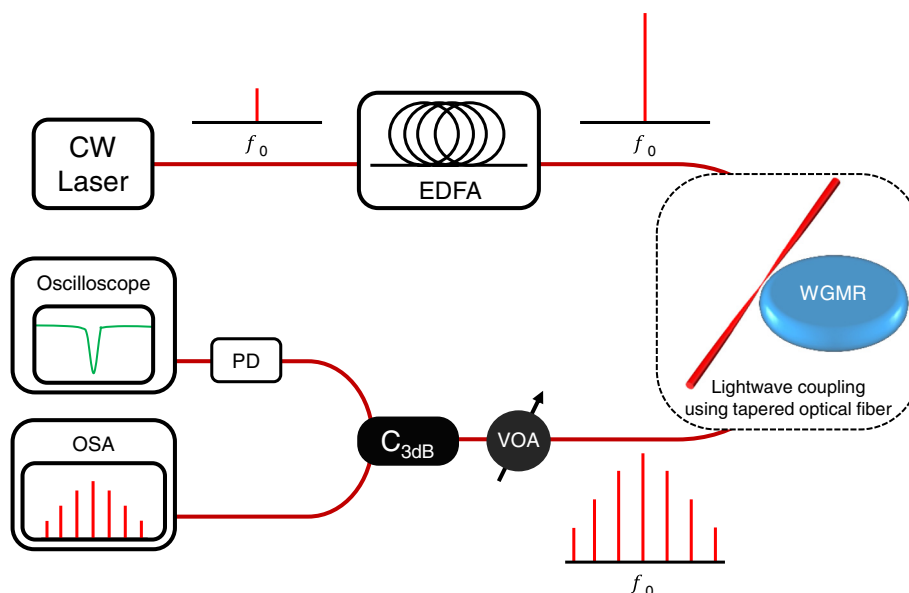
In this work, we investigate Kerr frequency comb generation in an overmoded MgF<sub>2</sub> whispering gallery mode (WGM) resonator. We achieve wide tunability in the Turing pattern regime when the resonator is pumped with fixed external pump power. We also discuss the thermal effects, as well as the asymmetric combs obtained owing to the influence of mode-crossing.

## 2 Experimental Setup

The experimental setup is illustrated schematically in Fig. 1. The WGM resonator used in this experiment is fabricated from a commercially available MgF<sub>2</sub> disk with a radius of ~6 mm. The disk is later centered and mounted on a home-made high-speed lathe. The edge of the disk is then carefully shaped and polished to create an optically smooth surface. The final disk features an intrinsic  $Q$  factor  $>10^9$  at 1550 nm. A tunable CW semiconductor laser with few

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**Fig. 1** Schematic view of the experimental setup. EDFA, erbium-doped fiber amplifier; WGMR, MgF<sub>2</sub> whispering gallery mode resonator; PD, photodetector; OSA, high-resolution optical spectrum analyzer; VOA, variable optical attenuator; C<sub>3dB</sub>, 3 dB 1 × 2 optical coupler.

kilohertz spectral linewidth is used as a pump source. The laser power is then further amplified by an erbium-doped fiber amplifier. The final pump power can reach a few hundred milliwatts. In order to evanescently couple light in and out of the cavity, a fiber taper with a micrometer-size waist is used. The throughput of the fiber is then connected to a 3 dB 1 × 2 fiber coupler. The split output signals are monitored with a photodetector and a high-resolution optical spectrum analyzer (APEX 2440B) separately.

### 3 Results and Discussion

#### 3.1 Primary Combs with Spacing from Gigahertz to Terahertz

The formation of single free spectral range (FSR) Kerr combs could involve many stages and produces wide bandwidth and multiple RF beatnotes.<sup>8,12,14</sup> Recent theoretical work has also shown that chaotic behavior of Kerr combs<sup>27,37,38</sup> exists when the pumping is sufficiently high. Primary Kerr combs or Turing patterns, as the first stage of the full comb, shows a strong phase-locked behavior compared with other combs.<sup>21</sup> Experimentally, we demonstrate primary combs generation with 4-FSR, 44-FSR, and 229-FSR spacing in the same resonator. The corresponding frequencies are 23.7, 260.0, and 1351.0 GHz as shown in Fig. 2.

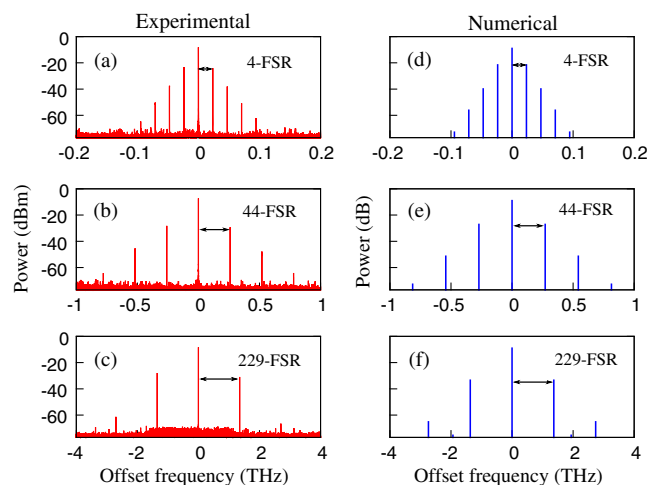
The theoretical model used to analyze comb generation here is a normalized Lugiato-Lefever equation with periodic boundary conditions<sup>30,32</sup>

$$\frac{\partial \psi}{\partial \tau} = -(1 + i\alpha)\psi + i|\psi|^2\psi - i\frac{\beta}{2}\frac{\partial^2 \psi}{\partial \theta^2} + F, \quad (1)$$

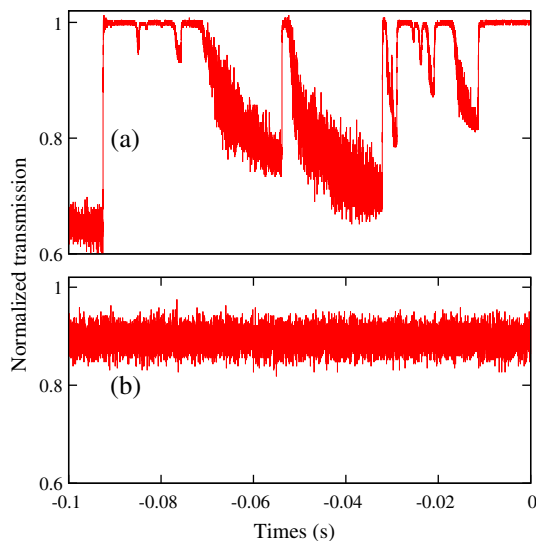
where  $\psi(\theta, \tau)$  is the total intracavity field in the moving frame,  $\theta \in [-\pi, \pi]$  is the azimuthal angle along the circumference of the disk, and  $\tau = \Delta\omega t/2$  is the dimensionless time, with  $\Delta\omega$  being the loaded mode linewidth. The dimensionless parameters in this equation are the frequency

detuning  $\alpha = -2(\omega_L - \omega_R)/\Delta\omega$ , which is the frequency detuning between the laser frequency  $\omega_L$  and the pumped mode resonance position  $\omega_R$ , the dispersion  $\beta = -2\zeta_2/\Delta\omega$ , with  $\zeta_2$  being the second-order Taylor coefficient of the eigenfrequency expansion and  $F^2$  being proportional to the external pump power.

In order to simulate similar primary combs, very different dispersion parameters are chosen for these three primary combs as shown in Fig. 2. It should be noted that the spectrum analyzer detects both the outcoupled intracavity field and the remaining pump field. This has to be taken into account when comparing the experimental and theoretical spectra. Although there are other different detuning and pump parameters to generate similar spectra, we believe that the main reason causing these significant different comb lines would be the various dispersion parameters in



**Fig. 2** Left column: the experimentally obtained spectra for three different primary combs with 4-FSR, 44-FSR, and 229-FSR spacing, respectively (from top to bottom). Right column: The corresponding numerical simulation. 4-FSR:  $\alpha = -0.2$ ,  $\beta = 0.4$ ,  $F^2 = 6$ ; 44-FSR:  $\alpha = -1$ ,  $\beta = 0.003$ ,  $F^2 = 6$ ; 229-FSR:  $\alpha = -1.2$ ,  $\beta = 0.0012$ ,  $F^2 = 6$ .



**Fig. 3** (a) Typical transmission spectrum with thermally broadened WGM resonances dips. The wavelength of the pump laser is increasing with time. (b) Transmission curve with fixed pump frequency where self-thermally stabilized state is achieved.

an overmoded resonator. These theoretical spectra are, however, in excellent agreement with those that can be obtained experimentally.

### 3.2 Thermal Noise and Kerr Combs

Figure 3(a) shows a typical thermally broadened WGM resonance spectrum in the throughput of the fiber taper. It results from the so-called thermal bistability in ultrahigh  $Q$  resonators.<sup>39</sup> In this experiment, the CW pump laser was detuned from the blue side of the resonances. Thus, the positive thermal coefficient of refractive index and thermal expansion then cause the red shift of the cold cavity resonance when it is continuously heated up by the pump laser. This feature has also been reported for the use of fast micro-laser characterization.<sup>40</sup> On the other hand, rich mode structures can be clearly seen in Fig. 3(a). It is expected that different families of modes feature different mode volumes,  $Q$  factors, and coupling efficiencies. As a result, they usually experience different thermal behaviors as shown in Fig. 3(a). There have been efforts to achieve single-mode ultrahigh  $Q$  WGM resonators.<sup>18,41</sup> However, this task proves to be very difficult.

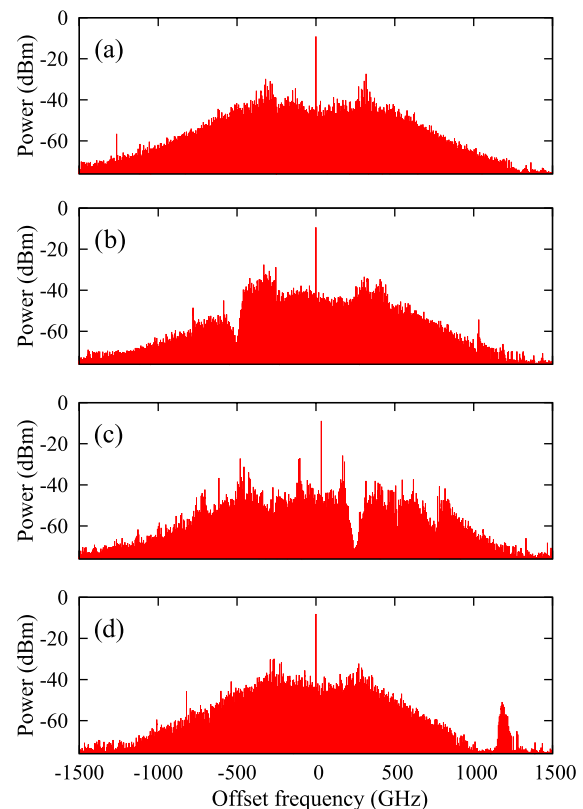
As one can see, noise structures are observed on the blue side of the resonances. With increasing pump wavelength, the heating causes the shift of the resonance in the same direction. It slows down the relative detuning on the blue side of the resonance and makes the noise structure observable. On the red side of the resonance, the decreased coupled pump power no longer can overcome the heat dissipation. The temperature starts to decrease and, thus, the resonance shifts in the opposite direction of the laser scan, which leads to a strongly reduced scan time for this side. It should be noted that these noises are easily missed when advance functions like averaging or high resolution in the oscilloscope are used. Noises have been seen even without Kerr comb generation as shown in Fig. 3(b). When the pump laser is coupled into an optical resonance with a very narrow linewidth, a small relative frequency fluctuation between the

pump and the resonance frequency will be transformed into the amplitude noise and can be easily observed. Pump laser power, frequency, and cavity resonance jitter can result in this noise. The latter one is usually related to the fluctuation of the environmental temperature, the heating, and the Kerr shift induced by the absorbed pump laser. These fluctuations can be substantially attenuated using well-known laser-locking techniques.

An example of the thermal noise in the transmission spectrum with fixed pump frequency is shown in Fig. 3(b). We manually decrease the frequency of the pump laser until a WGM is excited, symbolized by a clearly increased noise amplitude. In each step, we need to wait a few seconds for the diffusive cooling and laser heating to reach an equilibrium. This timing depends on the disk size,  $Q$  factors, and the detuning step. Beside the eventual chaotic characteristics of the comb,<sup>27,37,38</sup> we believe that the thermal noise shown here is one of the main reasons why RF beatnote linewidths are of the order of (sub-)megahertz level when using free running pump laser with the self-thermally locking technique. However, this problem can be overcome by using PDH technique to lock the pump laser frequency to the corresponding optical modes. It has recently been successfully demonstrated and results in low phase noise RF beatnotes with the linewidth in near hertz level.<sup>23</sup>

### 3.3 Mode-Crossing Effects on Kerr Combs

In an ultrahigh  $Q$  resonator coupled with a fiber taper, very complicated mode structures can be easily observed. Here, we put the taper in contact with the resonator to further excite higher-order modes and increase the mode-crossing



**Fig. 4** Spectra of symmetric and asymmetric Kerr frequency combs with single-FSR spacing when arbitrary WGMs are pumped.



probability. Figure 4 presents a Kerr comb spectra obtained when arbitrary modes are pumped. In comparison, a symmetric comb spectrum is presented in Fig. 4(a). As can be seen, Figs. 4(b) and 4(c) show asymmetric combs with locally weakened comb lines in either smaller or larger frequency regimes within 500 GHz offset from the pump. The enhanced comb lines are also observed in Fig. 4(d). These effects could result from the mode-crossing and possibly higher-order dispersion.<sup>19,42</sup> The small spectra spanning range could result from the high-order mode excitation and low coupling efficiency.

#### 4 Conclusion

In conclusion, we have shown that a generation of primary combs with line spacings ranging from gigahertz to terahertz in a single resonator is possible. It is known that it is generally difficult to obtain primary combs with low multiple-FSR spacing ( $<10$ ). The present work thereby proves that combs with a stable frequency spacing of 24 GHz can be generated, thereby allowing the achievement of ultrastable microwave generation in frequency bands of interest for aerospace technology. We have also presented experimental evidence of thermal locking effects on various families of modes, as well as the asymmetric combs that arise from a mode-crossing effect in an overmoded resonator. Future work would include a necessary pump-resonance locking technique to avoid thermal noises and better study of Kerr combs for low phase noise microwave generation.

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