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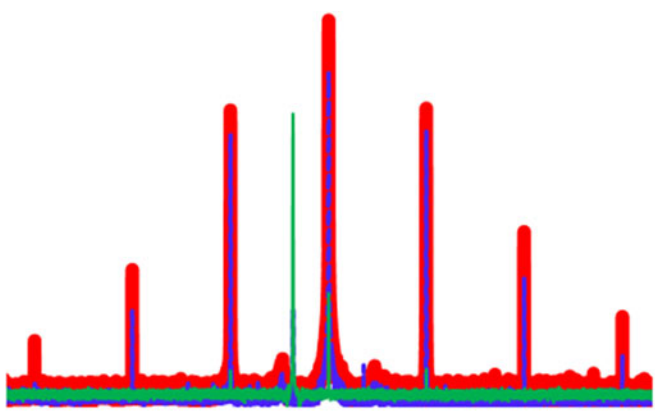

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Optimization of Close-In Phase Noise for Microwaves Generated With Kerr Combs Using Brillouin-Assisted Pump Depletion

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Abstract: We report a method to improve the close-in phase noise performance of microwave signals generated using Kerr optical frequency combs. We propose a technique based on Brillouin-assisted pump depletion, and we show that the resulting phase noise close to the microwave carrier can be significantly lowered. We consider microwaves with 18 and 30 GHz frequency and achieve phase noise improvements up to 10 dB for close-in phase noise (offset <10 kHz). This technique targets applications where particularly high spectral purity is required at sub-kHz offset from the carrier frequency.

Index Terms: Microwave photonics, whispering gallery mode resonators, oscillators, phase noise.

1. Introduction

Ultra pure microwaves are currently needed in a wide range of applications, ranging from navigation, time-frequency metrology, sensing, as well as aerospace and telecommunication engineering. In recent years, microwave photonic systems have arisen as interesting alternatives as they help transfer coherently the exceptional spectral purity of lasers down to the microwave spectral range, while providing, at the same time, a natural interface between optics and electronics.

In particular, Kerr optical frequency combs are considered one of the most promising technique to provide ultra-pure microwave signals [1], [2]. These combs are obtained by pumping an ultra-high Q whispering gallery mode (WGM) resonator with a resonant pump laser. A cascaded four-wave mixing process transfers the pump photons to the neighboring azimuthal eigenmodes, thereby leading to the formation of a set of evenly spaced spectral lines in the frequency domain, that is, to the so-called Kerr optical frequency comb [3]–[5]. This comb generation process critically depends on the strength of the Kerr nonlinearity, the overall group velocity dispersion, the cavity losses, and, finally, the laser pump power and frequency [6]–[17]. Even though the pump frequency is of the order of 200 THz (corresponding to 1550 nm), the free-spectral range of these resonators typically ranges from 1–100 GHz, and these microwave intermodal frequencies can be retrieved through demodulation with a large bandwidth photodiode. Kerr combs have already achieved remarkable performances in the areas of aerospace and telecommunication engineering, amongst other applications [18]–[30]. The specific advantages of these combs are mainly the potentially small size and weight payload, as well as the small energy footprint.

As far as ultra-pure microwave generation is concerned, a major metric is the phase noise performance at a given frequency offset from the carrier. Most studies do monitor the phase noise performance at 10 kHz offset from carriers in the GHz range. However, many applications are concerned with smaller frequency offsets. For example, the frequency shift Δf in the echo probe from Doppler radars operating at microwave frequency f is proportional to the radial velocity v of the target, following $\Delta f/f = 2v/c$, where c is the velocity of light in vacuum. Therefore, the detection of targets moving with very small radial velocity induces an accordingly small Doppler shift, which can be resolved by the radar only when the closed-in phase noise of the probing microwave is low enough (for example, a target radially moving at 3 m/s induces a Doppler shift of only 200 Hz when probed by a microwave at 10 GHz). Hence, achieving the lowest possible phase noise in the 100 Hz–10 kHz range is critical for such applications (see an early discussion in [31]). This requirement is conflicted by the fact that phase noise always increases significantly in this frequency range. Indeed, as the offset is reduced, it is known that phase noise increases at least by 20 dB/decade in presence of white noise and up to 30 dB/decade when there is $1/f$ noise. It is therefore important to investigate techniques that are able to improve the close-in phase noise of RF signals generated by Kerr optical frequency combs, in order to increase its potential for a wider set of applications where ultra-low noise is needed at sub-kHz offsets.

For this purpose, here, we report a technique relying on Brillouin-assisted pump depletion, which selectively reduces the high power level of the laser lightwave, thus preventing early saturation of the photodiode. Indeed, one of the problems affecting the close-in phase noise is the large power gap between the laser lightwave and the other comb teeth. This leads to a rapid saturation of the photodiode used for the conversion of the optical comb into the microwave frequency domain by the high power laser lightwave. The conversion process is therefore affected and the phase noise of the generated signals in the RF domain is therefore degraded. This technique is based on the use of the stimulated Brillouin scattering (SBS) narrow gain to selectively deplete an optical carrier and thereby increase the modulation depth in a microwave optical link [32]. Stimulated Brillouin scattering is recognized as the dominant optical fiber nonlinearity and is caused by the interaction between a light and acoustic waves, creating a backscattered Stokes wave which is frequency downshifted from that of the incident lightwave by an amount $\nu_B \simeq 11$ GHz in silica fibers [33]. Indeed, the Brillouin gain profile has a full width at half maximum (FWHM) bandwidth $\Delta\nu_B = 20$ MHz in typical silica SMF28 fibers. Such a narrow gain spectrum is therefore very selective regarding the different generated intermodal frequencies for the combs, which are in the GHz range in our case.

Above a certain threshold level for the incident optical signal, the backscattered Stokes wave becomes more intense and the incident lightwave power depletion occurs. This is due to the fact that the increased portion of the incident optical power injected in the fiber is converted into the backscattered Brillouin Stokes wave. When a Kerr comb is launched in a few km-long optical fiber, the pumped (or central) mode will undergo pump depletion if above threshold, while the sidemodes will remain unaffected for having a sub-threshold power. As a result, the pump-to-sidemode power ratio is reduced and the phase noise performance of the generated microwaves is improved for sub-kHz offset frequencies.

In the forthcoming sections, we present in detail the experimental setup under study. We then analyze and discuss the close-in phase noise performance of the system, and the last section will conclude the article.

2. Experimental Setup

The experimental setup allowing for a selective laser power depletion technique using stimulated Brillouin scattering is depicted in Fig. 1. The WGM resonator is made with a MgF_2 crystal with 12 mm diameter, corresponding to a free-spectral range of 6 GHz. It was manufactured in our laboratory using the grinding and polishing technique [34]–[36]. This resonator featured a loaded

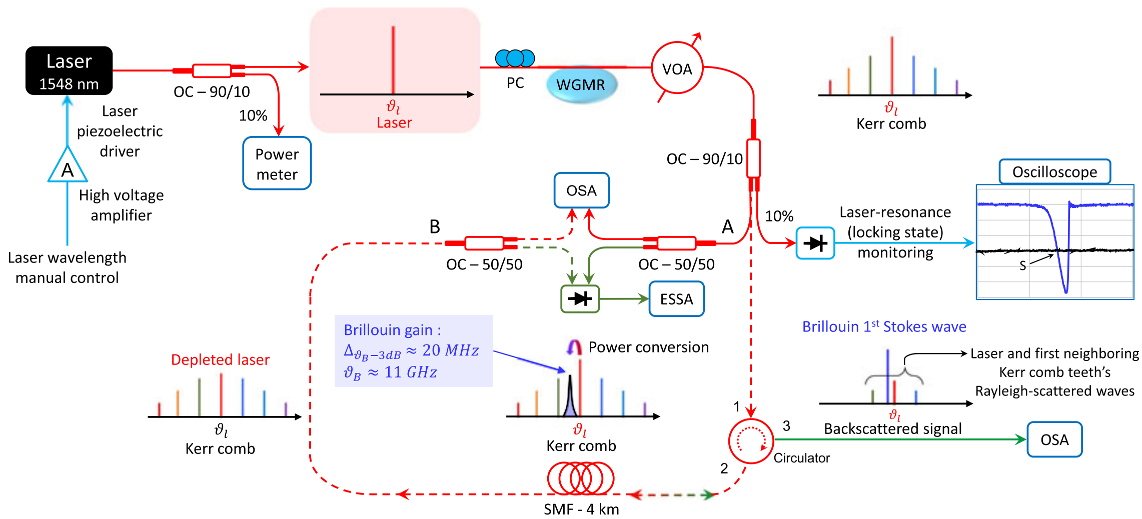

Fig. 1. Experimental setup for the selective laser power depletion technique using the stimulated Brillouin scattering. (Red) Optical path. (Dashed red) Optical path used when exploiting the Brillouin selective laser power depletion technique. (Green) RF path. (Blue) Low-frequency path for locking state S monitoring and control. PC: polarization controller; VOA: variable optical attenuator; OC: optical fiber coupler; OSA: optical spectrum analyzer; ESSA: electrical signal and spectrum analyzer; SMF: single mode fiber.

Q-factor of $\sim 10^8$ at 1550 nm, corresponding to a loaded linewidth $\sim$ 1 MHz. In this system, the laser power at the input microfiber (used to couple the lightwave into the WGM resonator) was set to 20 dBm and a polarization controller was used before the WGM resonator in order to optimize the eigenmode excitation. A laser-resonator add-only coupling configuration was implemented and the optical output at the through port of the WGM resonator was then attenuated using a variable optical attenuator (VOA). It was then split and 10% was sent to a slow photodiode followed by an oscilloscope in order to monitor and control the laser-mode locking state. The remaining 90% of the signal at point A was then split again and first sent simultaneously to an optical spectrum analyzer (OSA; APEX 2440B) and a fast photodiode (50 GHz u2t photodiode) followed by an electrical signal and spectrum analyzer (ESSA; Rohde & Schwarz FSW50) to be characterized.

Afterwards, in order to evaluate the efficiency of the Brillouin selective laser power depletion technique, the remaining signal was directly sent to an optical fiber circulator and the transmitted signal was sent to a 4 km single mode optical fiber (SMF) with transmission loss of 3.5 dB. The transmitted signal at point B was split and sent simultaneously to the OSA and the fast photodiode followed by the ESSA to be characterized. The backscattered signal part at the optical circulator third port was also sent to the OSA for characterization. Here, the proposed architecture involved a long optical fiber in order to further reduce the stimulated Brillouin scattering threshold. Nevertheless, this threshold should not be reduced too strongly in order to prevent the generation of a Brillouin comb and detrimental complexity in the optical spectra. It is important to note that our system does not rely on delayed feedback (like in fiber-based optoelectronic oscillators, for example, see [37]–[44]), since the signal at the output of the 4-km fiber is not coupled back to the comb generator.

In this experiment, the laser-mode locking state was first set manually by tuning the laser lightwave into an identified optical mode allowing for the generation of Kerr optical frequency combs. Once a thermal equilibrium was achieved, the laser lightwave remained into a stable locking state allowing the generation of a primary comb, characterized by a 30 GHz intermodal frequency. It should be noted that the so-called primary combs are characterized by multiple-FSR spacing in the frequency domain, and correspond to Turing rolls in the spatiotemporal domain (see [13] and [45]–[47]). This regime provides a significantly high degree of coherence, but if the resonator is further pumped beyond a certain limit, the intracavity field becomes chaotic [48], [49], and the coherence is partially lost [25].

One of the main problems encountered in the generation of RF signals using Kerr combs is the large power gap between the laser lightwave in the pumped optical mode and the other comb teeth. As a result, the photodiode converting the Kerr comb into the RF domain will be rapidly saturated by the high laser power and the performance of the generated RF signal is therefore degraded. The technique that we are proposing in the article is based on the use of the SBS narrow gain to selectively deplete the optical carrier and thereby reduce the power gap with the other comb teeth. Here, it should be mentioned that if an add-drop coupling configuration was used, the use of the comb at the drop port of the WGM resonator can provide a reduction in the laser power level and in the power gap between the laser signal and the neighboring comb teeth. This prevents the early saturation of the photodiode by the high power level laser lightwave, and therefore lead to higher RF power levels and better phase noise performance for the generated beat notes. On the other hand, a weak optical power coupling at the drop port of the WGMR must be used. This is in order to reduce the coupling loss and to prevent a degradation in the optical Q-factor of the WGM resonator, which would lead to an increase in the Kerr combs threshold and a degradation in their coherence performance. The use of stronger power coupling configuration is, however, needed to obtain better optical power levels at the drop port and, consequently, better power and phase noise results in the RF domain.

3. Phase Noise Analysis

The efficiency of the SBS laser power depletion technique has been first characterized using the comb featuring a 30 GHz intermodal frequency. The optical spectra of the comb were recorded before and after the 4 km SMF (i.e. for the split part of the signal at points A and B, respectively), and at the circulator third port for the backscattered signal [see Fig. 2(a)]. For the signal before the 4 km SMF, the measurements show high power level of the laser lightwave with a high power gap with the neighboring comb teeth ($\simeq$19 dB). On the other hand, the signal recorded after the SMF featured a low level laser lightwave with a low power gap with the neighboring comb teeth ($\simeq$12 dB). This proves that the high power laser lightwave exceeded the stimulated Brillouin scattering threshold inside the 4 km single-mode fiber and generated a backward-propagating Stokes wave. Consequently, the laser lightwave experienced a selective power depletion. This is visible in the power gap reduction by 7 dB that we obtained (the 3.5 dB loss induced by the SMF fiber also has to be taken into account here). Moreover, the optical spectrum of the backscattered part of the signal shows the generation of a relatively high power backscattered Brillouin Stokes wave, downshifted from the laser lightwave by 11 GHz. The other weak optical waves at the laser wavelength and the two first neighboring comb teeth wavelengths are due to the Rayleigh scattered waves for these comb components.

Despite the positive results obtained in the optical domain, the wide span RF spectra before and after the SMF have shown a low power RF signal at 30 GHz for the comb beat note after the SMF [see Fig. 2(b)]. A RF power difference of 22 dB is obtained between the beat notes detected before ($-$37 dBm) and after ($-$59 dBm) the SMF. This result is probably due to the high dispersion induced by the 4 km optical fiber and affecting the large intermodal frequency of the transmitted comb. Still, the measured narrow span RF spectra and the phase noise spectra of the beat note at 30 GHz of the comb show a better noise performance for the comb beat note after the SMF [see Fig. 2(c) and (d)]. The high noise floor in phase noise spectrum of the 30 GHz beat note after the SMF is due to the low RF output power, but it can be lowered using more powerful primary combs (which would yield higher levels for the RF output power).

In order to further confirm the efficiency of the stimulated Brillouin scattering laser power depletion technique, narrow span RF spectra and phase noise spectra of beat notes at 18 GHz for a second Kerr comb were recorded before and after the SMF [see Fig. 2(e) and (f)]. This second comb was generated in the same experimental conditions as the previous one but by pumping another optical mode of the WGM resonator, featuring similar characteristics of the previous optical mode. Here, an RF power difference of 9 dB is obtained between the beat notes detected before ($-$39 dBm) and after ($-$48 dBm) the SMF. Moreover, the results also show clearly better close-in phase noise

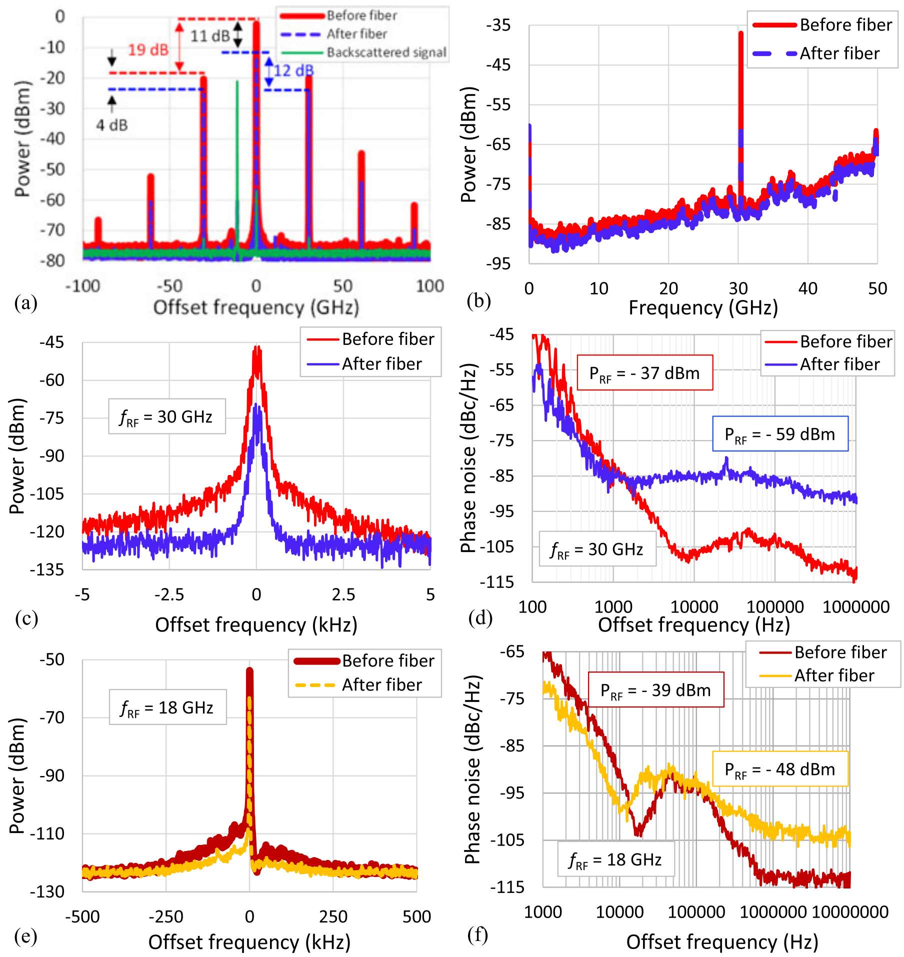

Fig. 2. Evaluation of the efficiency of the Brillouin selective laser power depletion technique, tested on two different combs generated using two different optical modes of the MgF_2 WGMR. (a) Optical spectra of the first comb recorded before and after the SMF and of the backscattered signal. (b) Wide span RF spectra of the first comb recorded before and after the SMF. (c) Narrow span RF spectra and (d) phase noise spectra of the beat note at 30 GHz of the first comb recorded before and after the SMF. (e) Narrow span RF spectra and (f) phase noise spectra of the beat notes at 18 GHz of the second comb recorded before and after the SMF.

performances performances for the comb beat note after the SMF. This improvement in the RF domain results compared to the first comb is probably due to the fact that the comb featuring an intermodal frequency of 18 GHz was less affected by the dispersion induced by the 4 km SMF than the comb featuring a large intermodal frequency of 30 GHz. In such case, the use of a zero dispersion optical fiber instead of the SMF could be a suitable solution for this problem, as the stimulated Brillouin scattering is also efficiently generated in such optical fibers.

4. Conclusion

In this paper, we have investigated a new technique to improve the close-in phase noise performance of microwaves generated with Kerr optical frequency combs. The technique is based on the use of the stimulated Brillouin scattering selective gain after the WGM resonator through port to selectively deplete the high power level of the central pumped mode of the comb when an add-only laser-resonator coupling configuration is implemented. We have achieved an improvement of up to 10 dB at sub-kHz frequency offsets.

Further research will explore the possibility of using a high-Q fiber ring resonator instead of the long optical fiber line in order to reduce the size of the system and significantly increase the efficiency of the stimulated Brillouin scattering [50]. This will also enable investigations into the limitation of such a technique in terms of power and phase noise. It is also known that both Brillouin and Kerr effects can be simultaneously excited in the same resonator [51]–[53], and an open question is to determine what is the effect of this intra-cavity stimulated Brillouin scattering as far as the phase noise performance of Kerr combs is concerned. Dispersion engineering for crystalline resonators [54], [55] will also offer the possibility to tune the intermodal frequency of the primary combs and, thereby, allow for the generation of a wide range of microwave frequencies using this scheme.

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